

Heterogeneous Agent Models in Continuous Time

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1 Model Description

Consider an economy with an infinite horizon and incomplete asset markets. Households face idiosyncratic income risk e_t and accumulate wealth a_t . Let $x_t = (a_t, e_t)$, I denote with $\psi_t(x_t)$ the joint distribution of the idiosyncratic states and with $f_t(x_t)$ the associated density function. Finally, $v_t(x_t)$ is the value function. I use the shorthand notation $m_a := (r_t a_t + w_t e_t - c_t)$, $m_e := \mu(e_t)$, $s_e := \sigma_e(e_t)$ for the drifts and standard deviation of the state variables. A competitive equilibrium in the HA model is given by $(c_t, da_t, K_t, L_t, r_t, w_t)$: Given prices r_t, w_t agents solve the HJB equation

$$\rho v_t = \max_{c_t} \left\{ u(c_t) + \frac{\partial v_t}{\partial a} m_a + \frac{\partial v_t}{\partial e} m_e + \frac{1}{2} \frac{\partial^2 v_t}{\partial e^2} s_e^2 + \frac{\partial v_t}{\partial t} \right\},$$
$$da_t = (r_t a_t + w_t e_t - c_t) dt,$$
$$de_t = \mu(e_t) dt + \sigma(e_t) dw_t.$$

Given prices K_t, L_t satisfies

$$F_K(K_t, L_t) = r_t + \delta,$$
$$F_L(K_t, L_t) = w_t.$$

Prices are such that the labor market, the asset market, and the goods market clear:

$$\frac{\partial f_t}{\partial t} = -\frac{\partial}{\partial a}(f_t m_a) - \frac{\partial}{\partial e}(f_t m_e) + \frac{1}{2} \frac{\partial^2}{\partial e^2}(f_t s_e^2),$$
$$L_t = \int_X e_t d\psi_t = \int_X f_t(x_t) e_t dx_t,$$
$$K_t = \int_X a_t d\psi_t = \int_X f_t(x_t) a_t dx_t,$$
$$\int_X c_t d\psi_t + I_t = F(K_t, L_t).$$

2 Solution Methods

I solve the model in sequence space using the finite difference approach of [Achdou, Han, Lasry, Lions, and Moll \(2022\)](#) and the model's Jacobians as in [Auclert, Bardóczy, Rognlie, and Straub \(2021\)](#). Here, I only provide an illustrative example. The lecture notes on this website contain further details on these models and solution algorithms.

References

- Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll (2022). “Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach”. In: *Review of Economic Studies* 89, pp. 45–86.
- Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub (2021). “Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models”. In: *Econometrica* 89 (5), pp. 2375–2408.