Online Appendix for
“Energy Shortages and Aggregate Demand:
Output Loss and Unequal Burden from HANK”

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A Further details on the model

A.1 The wage Phillips curve

In this section I derive the wage Phillips curve of the model. The Hamiltonian associated to the wage setting problem with control $\dot{W}_{jt}$, state W_{jt}, and costate μ_t taking W_t, N_t, r_t as given is

$$H_t(\dot{W}_{jt}, W_{jt}, \mu_t) = \exp\left(-\int_0^t r_s ds\right)\left(\int_0^1 \frac{W_{jt}}{P_t}N_{jt} - \frac{\upsilon(N_{jt})}{u'(C_t)} - \frac{\Psi_w}{2}\left(\frac{\dot{W}_{jt}}{W_{jt}}\right)^2 N_t dj + \mu_t \dot{W}_{jt}\right).$$

The first order conditions are

$$H_{\dot{W}_{jt}} = -\Psi_w\left(\frac{\dot{W}_{jt}}{W_{jt}}\right)\frac{N_t}{W_{jt}} + \mu_t = 0,$$

$$H_{W_{jt}} = \left(\frac{1-\theta_w}{P_t} + \theta_w\frac{\upsilon'(N_{jt})}{u'(C_t)W_{jt}}\right)\left(\frac{W_{jt}}{W_t}\right)^{-\theta_w} N_t + \Psi_w\left(\frac{\dot{W}_{jt}}{W_{jt}}\right)^2 \frac{N_t}{W_{jt}} = r_t\mu_t - \dot{\mu}_t.$$

Imposing a symmetric equilibrium with $W_{jt} = W_t$ and $N_{jt} = N_t$, using $\dot{\mu}_t = \Psi_w(\dot{\pi}_{w,t}(N_t/W_t) + \pi_{w,t}\dot{(N_t/W_t)})$ in the second equation above, after simplifying and rearraging terms, yields the wage Phillips curve where $\mu_w := \theta_w/(\theta_w - 1)$. The price Phillips curve is isomorphic to the wage Phillips curve and its derivation follows the same steps.

A.2 Non-homothetic demand

This section briefly discuss the implementation of the household problem with non-homothetic preferences following the approach of Auclert, Rognlie, Souchier, and Straub (2021). In particular, rewriting the budget constraint as $da_t = (w_t z_t n_t + r_t a_t + d_t - \hat{c}_t - p_{e,t}\underline{c})dt$, we can solve the household problem for the net-of-subsistence expenditure $\hat{c}_t$. Then, the CES demand system presented in Section 2, specifies the composition of household expenditure.[1] The derivation of the CES demand system is standard and extremely similar to the derivation of the energy demand by firms presented in detail below in this appendix.

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[1] I find that since subsistence expenditure $p_{e,t}\underline{c}$ is a small fraction of C_t, the results are quantitatively identical if I use $\hat{C}_t$ instead of C_t to compute the marginal rate of substitution between labor and consumption.

A.3 Energy demand and marginal cost

In this section I derive the demand for the production factors, and an analytical expression for the marginal costs of the firms. In the model intermediate firms solve the following problem

$$\min_{E_{it},N_{it}} w_t N_{it} + p_{e,t} E_{it},$$

$$\text{s.t.} \quad Y_{it} = \left(\alpha^{\frac{1}{\sigma}} E_{it}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} N_{it}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

The first order conditions $w_t = mc_{it}(1-\alpha)^{\frac{1}{\sigma}} Y_{it}^{\frac{1}{\sigma}} N_{it}^{\frac{-1}{\sigma}}, p_{e,t} = mc_{it}\alpha^{\frac{1}{\sigma}} Y_{it}^{\frac{1}{\sigma}} E_{it}^{\frac{-1}{\sigma}}$ yields

$$N_{it} = (1-\alpha)\left(\frac{w_t}{mc_{it}}\right)^{-\sigma} Y_{it},$$

$$E_{it} = \alpha\left(\frac{p_{e,t}}{mc_{it}}\right)^{-\sigma} Y_{it}.$$

Substituting the demand functions in the cost function and taking the derivative with respect to Y_{it} delivers the real marginal cost of each firm, which is the same across firms and given by

$$mc_t = \left((1-\alpha) w_t^{1-\sigma} + \alpha p_{e,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

A.4 Decomposing the aggregate consumption response

In this section I provide further details on the derivation and implementation of the decomposition in Section 4.2, that follows the approach of Kaplan, Moll, and Violante (2018). In particular, totally differentiating aggregate consumption

$$C_t(\{p_{e,t}, r_t, w_t, d_t, N_t\}) = \int_X c_t(x_t; \{p_{e,t}, r_t, w_t, d_t, N_t\}) d\psi_t,$$

delivers a decomposition of the total consumption response given by

$$dC_t = \int_0^\infty \frac{\partial C_t}{\partial p_{e,s}} dp_{e,s} ds + \int_0^\infty \left(\frac{\partial C_t}{\partial w_s} dw_s + \frac{\partial C_t}{\partial N_s} dN_s + \frac{\partial C_t}{\partial r_s} dr_s + \frac{\partial C_t}{\partial d_s} dd_s \right) ds.$$

This expression gives the relative role of direct effect and indirect effects. The direct effect is given by the energy price in the first integral, the indirect effects are given by the changes in the labor market and financial markets in the second integral. Each term includes an interaction between a partial equilibrium response $\partial C_t / \partial x_t$ and general equilibrium changes dx_t. In practice, I compute each integral numerically by feeding the equilibrium path of $p_{e,t}, r_t, w_t, d_t, N_t$ one variable at a time while keeping all other variables fixed at the steady state.

B Additional results

B.1 Long-term energy shortages

In all the simulatons of Section 4.3 the energy shock lasts for 3 years and more than 70% of the energy shortfall is absorbed within 1 year. Here, I consider a more conservative scenario in which the energy supply shortage lasts for 5 years and less than 60% of the energy shortfall is absorbed within 1 year.[2] I begin using the basic model with flexible wages, energy consumption only for production, an energy decline of 10%, and $\sigma = 0.1$. I choose this value for the elasticity of substitution over the other cases discussed in Section 4.3 to build-in a dose of caution and at the same time acknowledge that the elasticity of substitution tend to be larger in the long run than the short run. Figure A.1 contrasts the consumption response under this new scenario with the consumption response to the energy shock in Figure 3. Now consumption falls by 2.7% and returns to its previous level only after 3 years. This implies that the cumulative loss can be substantial. However, the difference moslty reflects a larger decline in real wages relative to the baseline. The responses of the other variables are similar in both couterfactuals.

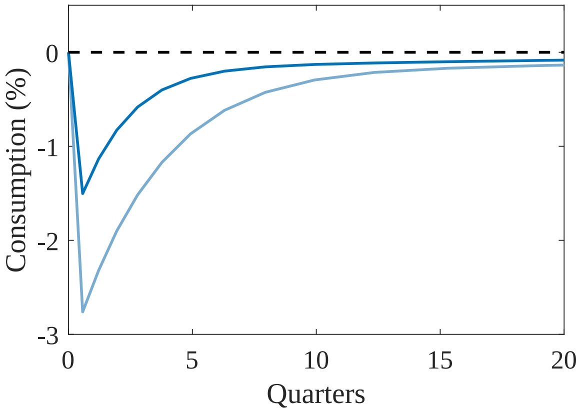

Figure A.1: Energy dynamics and aggregate consumption.

Note: The figure shows the percentage deviation from steady state of aggregate consumption in the high-persistence case (light blue line) and in the baseline case (blue line).

This exercise shows that the assumptions on the dynamic path of the supply shock are important for the aggregate outcomes. In the baseline calibration the income loss can increase by a factor of 1.8 under a highly persistent energy shock. On the other hand, the responses of inflation and energy prices are close to their baseline counterparts. However, despite the uncertainty regarding dynamic aspects of the energy shortfall the economic cost remains within the lower bound of the most pessimistic scenario of Section 4.3 with a fixed elasticity of 0.05.

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[2] In March of 2022 the European Commission announced a plan to cut out two-thirds of its Russian gas imports by the end of the year, and to make Europe independent from Russian fossil fuels well before 2030.

Since the long term consequences of the shock can be substantial I further investigate these effects in the baseline model with energy consumption by firms and households. As before I consider a case in which the energy supply shortage lasts for 5 years and less than 60% of the energy shortfall is absorbed within 1 year. Moreover, I also consider an intermediate case in which energy shortages last for 4 years. Throughout this experiment I use a low and fixed elasticity of substitution $\sigma = 0.1$. The analysis in this section is based on a sufficient statistic for the impact of an increase in the national energy expenditure from fossil fuels $B_t = p_{e,t}E_t$ on aggregate consumption. Specifically, I define the multiplier M_t such that

$$dC_t = M_t dB_t.$$

To measure the magnitude of the shock I use the energy expenditure as it contains information on both energy price changes and energy supply changes. Figure A.2 shows the response of consumption and energy expenditure share when the shock lasts respectively 3 years, 4 years, and 5 years. The energy expenditure share is defined with respect to annual GNI before the shock. In the baseline calibration the energy expenditure share increases to 9% of national income, a very large increase in the total energy bill of the economy. It is important to highlight that while energy consumption is complementary for the consumption of other goods, households can partly substitute energy use. These effects offset the impact of a large increase in the total energy bill on income. If the shock lasts for 5 years the expenditure share rises to 8% of national income and remains above the steady state level for 3 years leading to a consumption loss of 2%. In the baseline case the cumulative consumption loss over the first year is 2.3% while if energy shortages last for 5 years the cumulative loss over the first year is 5.3%. Figure A.2 shows that most of the consumption loss occurs in the first year after the shock. Moreover, the elasticity of substitution increases over time. For these reasons in my analysis I focus on the first year after the shock. There are two main reasons why the consumption loss is larger with fully flexible wages and energy consumption only by firms. First, with energy consumption by households the economic burden no longer falls only on firms and a substantial fraction of households are well-insured against income losses. Second, with nominal price and wage rigidities the drop of the real wages is more contained leading to a less severe reduction in households' earnings than in the economy with fully flexible wages.

To summarize the dynamic properties of the structural impulse response functions I use the cumulative multiplier given by

$$M_T = \int_0^T M_t dt.$$

Note that the multipliers in absolute value can be obtained from Figure A.2 using a simple back-of-the-envelope calculation, for example to compute the impact multiplier in the baseline case in which the energy shock lasts for 3 years one can use $(0.01/0.05) = 4M_0$ where M_0 is 0.05 and 4 is due to the fact that to compute the response of the energy share I use annual consumption.

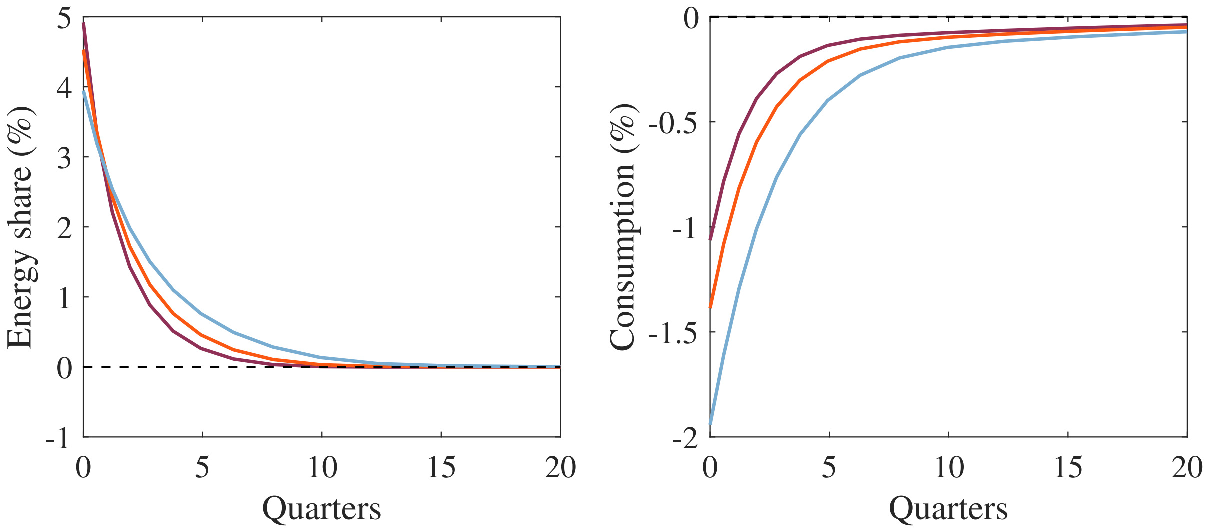

Figure A.2: Energy dynamics and the consumption multiplier.

Note: The figure shows the response of consumption and energy expenditure share when the shock lasts for 3 years (purple), 4 years (orange), and 5 years (light blue).

Table A.1: Structural multipliers

Shock Persistence	3 Years	4 Years	5 Years
Impact multiplier	0.05	0.08	0.12
1 Year multiplier	0.3	0.4	0.6

Note: The columns of the table report the multipliers when the energy shortages last for 3 years, 4 years, and 5 years. Multipliers shown in absolute value.

Table A.1 reports the impact multiplier and the cumulative multiplier over the first year and shows how these multipliers vary with the persistence of the energy shock. The cumulative multiplier is an order of magnitude higher than the impact multiplier and can be large if energy shortages last for 5 years. Moreover, the impact multiplier also increases with the persistence of the shock as households and firms anticipate today the long lasting effects of the shock. Overall, these computations quantitatively illustrate the dynamic implications of the aggregate demand effects for the output loss from an energy supply shock. I find that for a very persistent shock the dynamic amplification effects can be substantial. Moreover, these results suggest that the uncertainty regarding long lasting effects can amplify the economic losses. However, I find that even the cumulative multipliers are substantially below one.

B.2 A low-wealth calibration

In this section I investigate the sensitivity of the results to the size of the average MPC. In this exercise I use the basic model as in Section 4.3. The model generates large MPCs, yet these MPCs are at the lower bound of empirical estimates, as I discuss in Section 3. The average MPC shapes the demand amplification effect that I study in this paper. Therefore, in the spirit of the "liquid-wealth-only calibration" advocated by Carroll, Slacalek, and Tokuoka (2017), I analyze a low-wealth economy with a substantially higher average MPC. To achieve this I recalibrate the discount rate and the asset size to match a real return on wealth of 1% and an average quarterly MPC of 20%. This generates an economy with a large share of low-liquidity households, around 33% of the population. However, the quarterly MPC in the low-wealth calibration is 17% which is about two times the MPC of the baseline calibration.

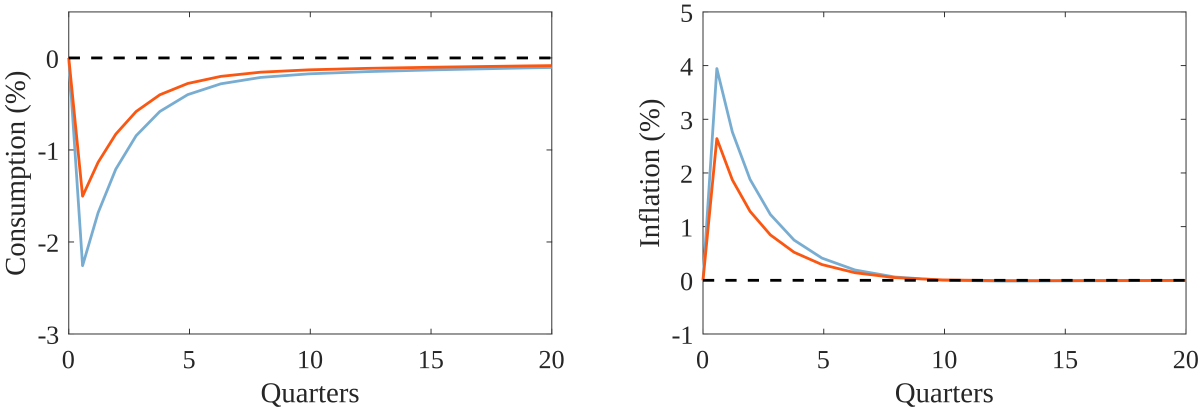

Figure A.3: MPC and aggregate consumption.

Note: The figure shows the percentage deviation from steady state in the low-wealth case (light blue line) and in the baseline case (orange line). $\sigma = 0.1$.

Figure A.3 shows the response of consumption and inflation to the energy shock across calibrations. At its peak the response of inflation increases from 2.6% to 4.4%. On impact, the response of the real interest rate rises from 2.6% to 4.9%, and the energy price increases by 30 percentage points across calibrations. Consumption falls by 2.2%, this implies that doubling the MPC only increases the consumption response by 0.7 percentage points or by 46 percent. This is somewhat a large change. However, it remains a small effect in comparison with the range of estimates generated by the uncertainty regarding the macro elasticity of substitution across inputs. Therefore, I leave a more complete analysis on the determinants of a large average MPC and the implications for the propagation of energy shocks to future work, instead I only emphasize here that the quantitative results in Section 4.3 are likely to be robust.

B.3 Labor supply elasticity

In this section I present the sensitivity of the results to the Frisch elasticity of labor supply $1/\nu$ in the baseline model. The value for the Frisch elasticity that I use in the baseline calibration is 1 this is in line with the estimates from Blundell, Pistaferri, and Saporta-Eksten (2016). However, since Chetty, Guren, Manoli, and Weber (2011) a value of 0.5 is more consistent with other micro estimates. Therefore, I set $\nu = 2$ without recalibrating the other parameters. Figure A.4 reports the responses of consumption, labor supply, inflation and energy price. The peak of the consumption response is 1.3%, while employment falls by 3%. The differences of aggregate consumption and employment from the baseline are within 0.3 percentage points. Real wages fall by 1.7% and the annualized real interest rate increases by 2.2%. Reducing the elasticity of hours to the real wage dampens the partial equilibrium response of employment. However, given a more contained adjustment of hours in general equilibrium this causes a larger fall in real wages even in the presence of sticky wages. Then, the demand amplification channel generates a deeper recession with a larger contraction of consumption and employment.

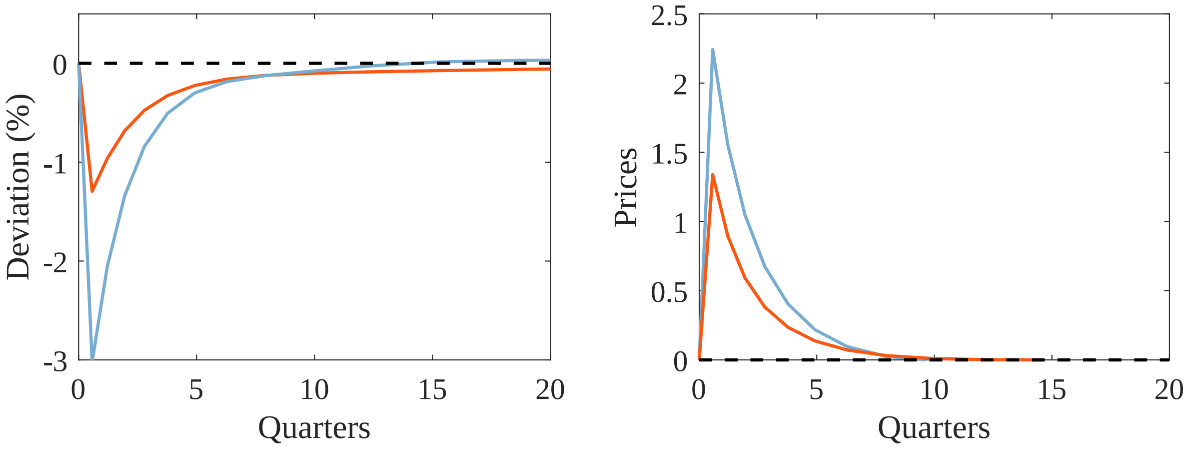

Figure A.4: Frisch elasticity and aggregate dynamics.

Note: The figures show the response of consumption (orange line left panel), labor supply (light blue line left panel), and inflation (light blue line right panel) in percentage deviations from steady state. Increase of the real energy price over its steady state value (orange line right panel) in decimals.

References

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